Difference in Interim Performance and Risk Taking

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Abstract

A growing empirical literature tests for tournament behavior in the mutual fund industry. The tournament hypothesis is built around the following intuitive idea: interim winners want to “lock in” their position and so decrease volatility of their portfolios, while interim losers increase the volatility in an attempt to catch up. We consider a simple rational model and show that, in equilibrium, interim winners choose more volatile portfolios than interim losers – contrary to the popular wisdom. Several recent studies present empirical evidence consistent with our model, but, based on the above intuitive argument, conclude against the tournament behavior. We argue that this conclusion is unwarranted.

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In an influential paper, Brown, Harlow, and Starks (1996) (BHS, hereafter) used the term “tournament” to describe the mutual fund market. The nature of this tournament is as follows. First, it is well-established that the inflows of new money into a fund depend on its relative performance via an increasing and convex flow-performance relationship (Chevalier and Ellison (1997), Sirri and Tuffano (1998)). Second, fund managers typically get a fixed percentage of assets under management as compensation. Combining these two features, it is natural to expect that fund managers care about relative performance as this directly affects their well-being, giving rise to a mutual fund tournament.

To test for tournament behavior, i.e. to see whether fund managers in fact respond to the incentives created by the flow-performance relationship, BHS suggest looking at how the managers change the riskiness of their portfolios over the second half of a year. According to BHS, mid-year losers are expected to increase the volatility to a greater extent than mid-year winners. The justification seems quite convincing: losers gamble in an attempt to catch up with the winners, while winners play it safe in order to protect their lead. Apart from having an intuitive appeal, the idea that losers adopt riskier strategies than winners is consistent with research on the economics of tournaments in various settings (Ehrenberg and Bognanno (1990), McLaughlin (1988)) and also with “gambling on resurrection” by troubled banks, who in our terminology can be thought of as interim losers (Dewatripont and Tirole (1995)).

Is it possible that risk taking incentives of mutual funds differ from those in other environments? To address this question, we build a simple rational model of a mutual fund tournament. In the baseline model, we assume a large number (continuum) of risk-neutral fund managers with different levels of interim performance. Each manager can invest in two assets, a risky stock and a riskless bond, subject to short-sales constraints. The managers maximize the expected terminal wealth, which depends on the relative return through the flow-performance relationship. The main result of our paper is that,\(^1\) The main result of our paper is that,\(^1\)

\(^1\)In this paper, we take this relationship as exogenous. That is, we do not address the issue of why
in equilibrium, interim winners choose more volatile portfolios than losers. Hence, the tournament hypothesis that follows from our model is opposite to that proposed by BHS and subsequently widely used in the empirical work (Busse (2001), Qiu (2003), Goriaev, Nijman, Werker (2005), Reed and Wu (2005)). We then investigate a number of extensions of the baseline model to show that our result is robust.

Before explaining the intuition, let us first highlight the two important features that distinguish a mutual fund tournament from other tournament environments. The first is that the set of available strategies is common for mutual funds, i.e. all funds can invest in the same set of securities. As a result, a fund (e.g. an interim winner) can perfectly protect its interim lead against any other fund (e.g. an interim loser) by investing in the same portfolio as the latter.\(^2\) This is unlike, say, football where one team’s strategy cannot be fully “hedged” by the opposite team. How does this invalidate the popular tournament reasoning? Simply, it is not necessarily optimal for a fund-loser to increase the riskiness of its portfolio by buying highly volatile stocks in case if a fund-winner already holds these stocks. If a loser is to have a chance of catching up, she should choose a portfolio which is different from that of a winner, rather than more volatile.\(^3\)

The second distinguishing feature of a mutual fund tournament is a large number of participants. In the literature on mutual funds, an “interim winner” is commonly defined as a fund whose mid-year performance is above the median, which implies that there are many (half the total number) interim winners. Hence, an interim winner can have many funds that are ahead in terms of interim performance. As a result, an interim winner may retail investors reward past performers despite the mixed evidence on performance persistence (see Carhart (1997), Bollen and Busse (2005), and references therein) and also on the link between past performance and stock-picking skills (Berk and Green (2004), Chen, Jegadeesh, and Wermers (2000)).

\(^2\)Interestingly, BHS seem to be partially aware of this issue when they mention that “... to the extent that they [winners] anticipate what those managers ranked below them might do, it may be necessary for them to increase risk as well, but they do not need to increase risk to the same extent as do the losers.” According to our model, however, winners rationally increase risk to a larger extent than losers.

\(^3\)A similar point is made in Basak, Pavlova, and Shapiro (2006) who show that a fund manager may optimally reduce her portfolio volatility when gambling against a relatively risky benchmark.
well try to improve its performance even further. This is different from football where, with only two teams playing, a team which is ahead cares more about preserving the current score than extending the lead.

From the above discussion, it follows that one should not \textit{a priori} expect the popular tournament argument to hold in the context of the mutual fund industry. While presenting this critique is one of the goals of this paper, the main goal is a constructive one – to propose a new tournament hypothesis that will have a theoretical underpinning. We show that, opposite to BHS’s informal reasoning, interim winners optimally choose a higher portfolio volatility than interim losers.

The intuition is as follows. The interaction of risk-neutrality and convexity of flow-performance relationship implies that a manager’s objective function is convex in performance. This leads to gambling behavior, whereby a manager maximizes the volatility of her tracking error, the difference between own and industry-average returns. Hence, an individual manager will invest in only one asset, either risky or riskless. However, the aggregate industry portfolio cannot, in equilibrium, consist of one asset. Indeed, in this case each manager would have incentives to choose the other security so that to maximize the tracking error volatility. Hence, in equilibrium it must be the case that some managers invest in the risky stock, while the others invest in the riskless bond.

Why it is the interim winners who choose the risky stock? Since a fund’s year return is a product of the interim return and the return over the second half of the year, a fund’s high level of interim performance translates into a high level of year-end return volatility \textit{if} the manager invests in the risky stock. Indeed, the return volatility is equal to interim performance squared multiplied by the volatility of the asset chosen in the mid-year. If on the other hand an interim winner invests in the riskless bond, then all volatility of the tracking error will be due to the volatility of the industry return, meaning that the interim performance factor is effectively “switched off”. Hence, interim winners invest in the stock
in order to “leverage” their high interim performance, thus driving interim losers into the bond.

Two characteristics of our model are worth commenting on. First, the framework is simplified and ignores many aspects of the real mutual fund industry. This is intentional: by introducing only one feature – relative performance considerations – into an otherwise standard one-period portfolio choice setting, we make sure that our results are driven purely by the tournament behavior. As a result, we provide a clean and parsimonious analysis of the question that has attracted considerable attention in the recent empirical work.

Second, for tractability we assume risk neutrality, which is not a common assumption in the portfolio choice literature. However, extending the baseline model to allow for heterogeneity in managers’ risk aversion does not change our conclusion (see Section III). Namely, for managers with low risk aversion the effect of interim performance on risk taking is the same as for risk neutral managers, which is to be expected. For relatively risk averse managers, the effect of interim performance on their risk taking is quantitatively small and ambiguous. Intuitively, the concavity of such managers’ objective function dominates the convexity of flow-performance relationship, implying that the incentives to win the tournament are not strong enough. Hence, these managers do not care much about their interim standings, which means that the effect of interim performance on the optimal portfolios is small. All in all, while the underlying mechanisms in a framework with risk aversion are somewhat different from the baseline case, the testable implication is exactly the same: interim winners (losers) are expected to increase (decrease) the riskiness of their portfolios. This is true as long as a non-trivial fraction of managers are relatively risk tolerant.

Our results point to the need to reassess the findings of the empirical literature on the mutual fund tournaments. In the first study of the topic, BHS provide evidence consistent
with their “intuitive” tournament hypothesis, and so against our model. However, Busse (2001) casts doubt on BHS’s results by showing that they depend crucially on the use of monthly data. To obtain a more precise volatility estimates, Busse uses daily returns and finds “no evidence that mid-year losers increase end of year risk more than winners. If anything, the results indicate the opposite.”

Qiu (2003) investigates mutual funds’ risk taking by employing a more recent sample than Busse and BHS. Similar to Busse, Qiu documents “the unexpected [our emphasis] result that mid-year loser funds have less incentives to increase their funds risk relative to mid-year winner funds,” and suggests some potential explanations, like termination risk or winner-takes-it-all phenomenon. While these features may well be part of the story, our analysis reveals that there is no need to resort to them in order to explain the empirical regularities – the findings of both Busse and Qiu are consistent with a simple rational tournament model.

The related literature includes Taylor (2003), Goriaev, Palomino, and Prat (2003), Li and Tiwari (2005), Loranth and Sciubba (2006). Similar to us, these studies investigate theoretically the effect of interim performance on portfolio volatility. However, they all consider settings with a small number (mostly two) of funds. It is not at all clear that the ensuing predictions are applicable to the actual mutual fund industry comprised by hundreds of funds. Another shortcoming of these two-fund models is that they provide no guidance for empirical work regarding how to divide funds into interim winners and losers. The widely used approach when the fund with median interim return serves as the threshold is pretty much ad hoc. In our model, the threshold fund is derived endogenously.

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4Goriaev, Nijman, and Werker (2005) argue that monthly data is more robust to autocorrelation of the daily returns. Nevertheless, they reach a similar conclusion to Busse regarding funds’ risk taking behavior once they account not only for auto- but also for cross-correlation of returns.

5Basak and Makarov (2006) also consider a framework with two funds, however their focus is not on addressing the tournament hypothesis but on understanding the strategic interaction among a small number of top-performing funds.

6To give one example of why the two-fund assumption is not innocuous, consider Taylor (2003) who obtains an equilibrium where each of the two funds uses a mixed strategy. This result is a direct consequence of the strategic nature of the funds’ interaction whereby each fund is trying to “confuse” the opponent. It is doubtful that such economic mechanism may be at work in reality where each fund is surrounded by many competitors.
This is important because, as discussed at the end of Section II, an empirical researcher who uses the median fund as a threshold may, in some scenarios, erroneously reject the tournament hypothesis. Palomino (2005) considers a tournament with an arbitrary number of mutual funds but he does not investigate how interim performance affects funds’ optimal portfolios, which is the main focus of our paper. Also related is work on tournaments in other economic settings (Lazear and Rosen (1981), Green and Stokey (1983), Bhattacharya and Guasch (1988), Taylor (1995), Zwiebel (1995), among many others).

Our paper also contributes to the literature that investigates the effect of convexities in managers’ objective functions on the optimal portfolio choice. Examples are studies by Grinblatt and Titman (1989), Carpenter (2000), Ross (2003), Cuoco and Kaniel (2006), Basak, Pavlova, and Shapiro (2007), Panageas and Westerfield (2007). Interestingly, a common feature shared by most of this literature, including our paper, is that the obtained results are seemingly counterintuitive. Given this pattern, it seems fair to argue for caution in relying on common wisdom in settings where managers’ compensation is (locally) convex.

The paper proceeds as follows. Section I describes the baseline economy. Section II presents the equilibrium analysis. In Section III, we discuss the plausibility of our assumptions, and also provide several extensions to investigate the robustness of our results. Section IV concludes. The Appendix contains all proofs.

I Economy

There is a continuum of funds in the economy, indexed by $i \in [0,1]$. Hereafter, we use the terms “fund”, “manager”, and “fund manager” interchangeably. Fund managers are assumed to be risk-neutral.\footnote{In Section III, we discuss the extension to the case of risk-averse managers.} Financial investment opportunities are given by a riskless
bond and a risky stock, both in perfectly elastic supply. For simplicity, we assume no
discounting. The return on the riskless asset is normalized to 1. The return on the risky
asset, denoted by \( x \), is normally distributed with mean \( \mu \) and variance \( \sigma^2 \). We make
the expected returns on the stock and the bond equal and set \( \mu = 1 \). This ensures that
managers’ choice between the stock and the bond is driven purely by considerations of
volatility.\(^8\)

There are two time periods: \( t = 1 \) and \( t = 2 \), corresponding to mid-year and year-end,
respectively.\(^9\) At time 1, manager \( i \) starts with a certain level of interim performance,
denoted by \( r^0(i) \). Each manager knows interim performances of all the other managers.
We assume that \( r^0(i) \) is increasing in \( i \).\(^10\) Given our focus, we do not address why funds
have different mid-year performances.\(^11\) Denote by \( \alpha(i) \) the fraction of manager \( i \)’s wealth
invested into the risky stock, so \( 1 - \alpha(i) \) is invested into the bond. We require that
\( 0 \leq \alpha(i) \leq 1 \) for all \( i \in [0, 1] \) to reflect no short-sales constraint.\(^12\)

The absolute performance of manager \( i \) at \( t = 2 \), denoted by \( R(i) \), is given by \( R(i) =
r^0(i)r^1(i) \), where \( r^1(i) \) is manager \( i \)’s return between time 1 and 2. Denoting by \( x \) the
realization of the stock return, we have

\[
r^1(i) = \alpha(i)x + (1 - \alpha(i)).
\] (1)

\(^8\)In Section III, we analyze the case \( \mu > 1 \).
\(^9\)Year-end is an important date since around this time many popular fund rankings are published in
the media, and based on them households choose funds for investing money.
\(^10\)It means that managers are sorted based on interim performance, with manager 0 having the lowest
interim return, \( r^0(0) > 0 \), and manager 1 having the highest, \( r^0(1) \).
\(^11\)Clearly, the reason for such heterogeneity in interim performance is that managers have been following
different investment policies in the first half of the year, which can be due to difference in (among other
factors) risk aversions, time horizons, available information.
\(^12\)In Section III, we discuss the plausibility of this assumption in the context of mutual funds.
The industry-average absolute performance, denoted by $\bar{R}$, is defined as

$$\bar{R} = \int_0^1 R(i)\,di = \int_0^1 r^0(i)r^1(i)\,di. \tag{2}$$

Proposition 1 characterizes the distribution of $\bar{R}$.

**Proposition 1** For a given set of portfolio strategies $\alpha(i)$, the industry-average return $\bar{R}$ is given by

$$\bar{R} = r^0(\bar{\alpha}x + (1 - \bar{\alpha})), \tag{3}$$

where

$$\bar{\alpha} = \frac{\int_0^1 r^0(i)\alpha(i)\,di}{\int_0^1 r^0(i)\,di}, r^0 = \int_0^1 r^0(i)\,di. \tag{4}$$

As follows from (3), one can think of $\bar{R}$ as the return on the fund with interim performance $\bar{r}^0$ and fraction of wealth $\bar{\alpha}$ invested in the stock, where $\bar{\alpha}$ and $\bar{r}^0$ are given in (4).

The relative performance of manager $i$ is defined as the difference between her own and industry-average absolute performances. Motivated by empirical evidence, we assume that the inflow of new investments into fund $i$ is an increasing and convex function of its relative performance, $f(R(i) - \bar{R})$, where $f'(\cdot) > 0, f''(\cdot) > 0$. Managers are assumed to know exactly the functional form (common for all funds) of $f(\cdot)$.

For notational simplicity, we assume that at the beginning of the year, i.e. prior to time 1, all managers start with unit wealth. This is without loss of generality, since year-start wealth is constant (as of time 1) and so does not affect the optimization. Then, the
terminal wealth of manager $i$ at time 2, denoted by $W(i)$, is given by

$$W(i) = R(i) + f(R(i) - \bar{R})$$

$$= r^0(i)(\alpha(i)x + 1 - \alpha(i)) + f(r^0(i)(\alpha(i)x + 1 - \alpha(i)) - \bar{r}^0(\bar{\alpha}x + 1 - \bar{\alpha})).$$

(5)

Because individual managers are atomistic, they take the distribution of $\bar{R}$ as given when choosing the optimal portfolios, which, from Proposition 1, is equivalent to taking $\bar{\alpha}$ as given. Henceforth, an equilibrium in our economy is defined as follows.

**Definition 1** An equilibrium is a pair $(\alpha^*(i), \bar{\alpha}^*)$ such that

(i) **Best Responses.** Given $\bar{\alpha}^*$, $\alpha^*(i)$ is a solution to manager $i$’s maximization problem

$$\max_{\alpha(i) \in [0,1]} E[W(i)]$$

for all $i \in [0,1]$, where $W(i)$ is given in (5).

(ii) **Aggregation.** Aggregating over individual portfolio strategies $\alpha^*(i)$, as in (4), yields $\bar{\alpha}^*$:

$$\bar{\alpha}^* = \frac{1}{\bar{r}^0} \int_0^1 r^0(i)\alpha^*(i)di.$$  

(7)

**II Equilibrium**

In this Section, we solve for an equilibrium in the economy described in Section I.

First, we characterize the managers’ best responses, denoted by $\hat{\alpha}(i)$ for manager $i$. For a given $\bar{\alpha}$, manager $i$’s best response is the solution to (6). Looking at (5), notice that the first term, $r^0(i)(\alpha(i)x + 1 - \alpha(i))$, drops out when we maximize the expected terminal wealth because the expected returns on the stock and bond are assumed to be equal,
$E[x] = 1$. Hence, in line with the earlier discussion of this assumption, assuming $E[x] = 1$ isolates the tournament behavior since the managers’ optimal policies are determined only by the “tournament” term $f(\cdot)$ in (5). To proceed, we need to specify the functional form of $f(\cdot)$. We assume that

$$f(R(i) - \bar{R}) = \exp(R(i) - \bar{R}),$$

which will allow us to characterize the equilibrium in closed-form. Proposition 2 presents the managers’ best response policies.

**Proposition 2** For a given $\alpha^\ast$, manager $i$’s best response is given by

$$\hat{\alpha}(i) = \begin{cases} 
1, & r^0(i) \geq 2\alpha^\ast \bar{r}^0 \\
0, & r^0(i) < 2\alpha^\ast \bar{r}^0 
\end{cases}$$

(8)

From Proposition 2, there exists a threshold value of interim performance, $2\alpha^\ast \bar{r}^0$, which divides all funds into two categories. Those with interim performance above the threshold — interim winners — invest fully into the risky stock. The rest — interim losers — hold only the riskfree bond. This result is opposite to the tournament hypothesis widely used in the empirical literature.

The intuition is as follows. The interaction of risk-neutrality and convexity of the flow-performance relationship implies that managers are effectively risk loving, seeking to maximize the volatility of the tracking error. Since a fund’s year return is a product of the interim return and the return over the second half of the year, a fund’s high level of interim performance translates into a high year-end return volatility if the manager invests in the risky stock. If, on the other hand, an interim winner invests in the riskless bond, then all volatility of the tracking error will be due to the volatility of the industry return, meaning that the interim performance factor is not at work. Hence, it is optimal
for interim winners to “leverage” their high interim performance by investing in the risky asset. For interim losers, the interim performance factor is low, and so it is optimal to “switch it off” by investing in the bond only.

Having described the best responses, we can now fully characterize an equilibrium in our economy.

**Proposition 3** An equilibrium aggregate risky investment $\bar{\alpha}^*$ is implicitly given by

$$\bar{\alpha}^* = \frac{\int_{i^*} r^0(i)di}{\bar{r}^0},$$

(9)

$$r^0(i^*) = 2\bar{\alpha}^*\bar{r}^0.$$ (10)

The solution always exists and is unique. The corresponding managers’ equilibrium strategies $\alpha^*(i)$ are obtained by substituting $\bar{\alpha}^*$ into the best responses (8).

Proposition 3 reveals that the division of funds into two categories, winners and losers, depends on the functional form of $r^0(i)$, i.e one needs to know the distribution of the interim returns across funds. This result casts doubt on the methodology commonly employed in empirical papers where the median fund serves as a threshold dividing winners from losers. Unlike ours, the existing theoretical tournament models with only two funds are silent on how to choose the threshold. This issue may be important for empirical research. For example, suppose that the true set of interim winners consists of the top quarter of mutual funds. A researcher who, following the standard approach, combines the funds above the median into one group may fail to find a significant change in volatility for this group, and so mistakenly reject the tournament behavior. Indeed, one half of the group are actually interim losers, adjusting the riskiness of their portfolios in the opposite direction from winners. Our model with continuum of mutual funds addresses to some extent this concern by endogenously deriving the threshold fund.
III Extensions and Discussion

In this Section, we present several extensions of the base model in order to investigate the robustness of our central result, and also discuss the plausibility of some of the assumptions.

A Risk Aversion

In the base model of Section I, we assume that fund managers are risk-neutral. Combined with the convexity of the flow-performance function, this implies that the managers are effectively risk-loving since their objective functions are convex in performance.\textsuperscript{13} It turns out that it is this convexity of the resulting objective function that is important for our results, rather than risk-neutrality \textit{per se}. To illustrate this point, below we present a numerical example.\textsuperscript{14}

\textit{Example 1.} Suppose managers are risk-averse with CRRA utility function

\[ u(W) = \frac{W^{1-\gamma}}{1-\gamma} \]

Given this utility function, we have to change the stock’s distribution so that to disallow negative wealth. Specifically, we assume that the stock return is uniformly distributed over \([0.9, 1.1]\). Other assumptions used in this example are: \(\gamma = 0.9, f(\cdot) = \exp(2(R(i) - \bar{R}))\), \(r^0(i) = 0.9 + 0.2 * i\).

\textit{Equilibrium in Example 1.} Solving the model numerically, we obtain that in equilibrium \(\bar{\alpha}\) is 0.081. In other words, approximately 8\% of the funds’ total wealth is invested in the

\textsuperscript{13}Note a somewhat interesting difference between ours and some other tournament settings. In many athletic tournaments, the players’ payoffs depend on the rank rather than, as in our case, on relative performance. As a result, the objective function of players is discontinuous, jumping upwards at the point when a player’s rank increases. This implies that the players’ objective functions exhibit local convexities. Importantly, such convexities, and hence the gambling behavior, are always present, \textit{regardless of how high players’ risk aversions are}. In our setting, the tournament behavior arises only when the flow-performance relationship strongly reward managers for high relative performance, i.e. when it is significantly convex.

\textsuperscript{14}We resort to numerical solution since the model is no longer analytically tractable when managers are risk averse.
stock, with the rest invested in the bond. Importantly, similar to the base model’s result, interim winners (losers) invest all their wealth into the risky (riskless) asset.

To better understand the role of risk aversion, we compute an equilibrium under the assumptions of Example 1, but with risk-neutral managers, $\gamma = 0$. In this case, we find that $\bar{\alpha}$ is 0.5. This is as expected: as risk aversion decreases, the risky asset becomes more attractive. Hence, a larger share of total wealth is put into the stock when $\gamma = 0$ compared to $\gamma = 0.9$ in Example 1.

To investigate the robustness, we have solved Example 1 for a number of other parameter values. The results concerning winners/losers risk taking are unchanged, as long as the risk aversion is low. In reality, it is likely that some fund managers are sufficiently risk averse so that their effective objective function remains concave even after accounting for the convexity of the flow-performance relation. As follows from our numerical simulations, for such managers there is no clear and economically significant effect of interim performance on portfolio volatility. The intuition is straightforward. Since risk aversion dominates tournament incentives, these managers do not care much about winning the tournament and, hence, about their interim standings.

To sum up, while the underlying mechanisms in a framework with heterogeneous risk aversion are somewhat different from the baseline case, the testable implication is exactly the same: interim winners (losers) are expected to increase (decrease) the riskiness of their portfolios. The only difference is that not all fund managers respond to the interim performance, but only the relatively risk tolerant ones.

From the above, it follows that the economic significance of tournament behavior depends on the number of risk tolerant managers. Addressing this question is a matter of empirical investigation, and so beyond the scope of our paper. However, there are some reasons to believe that fund managers are likely to have low risk-aversion. In his speech delivered before the United States Senate Committee on Banking, Housing, and Urban
Affairs, John C. Bogle, an active participant in the mutual fund industry for more than half a century, highlights:

The coming of the age of portfolio managers who serve as long as they produce performance moved fund management from the stodgy old consensus-oriented investment committee to a more entrepreneurial, free-form, and far less risk-averse approach. Before long, moreover, the managers with the hottest short-term records had been transformed by their employers vigorous public relations efforts, and the enthusiastic cooperation of the media, into stars, and a full-fledged star-system gradually came to pass.\textsuperscript{15}

\section{B \hspace{1em} Multiple Stocks}

Suppose that the investment opportunities are represented by $N > 1$ uncorrelated risky stocks.\textsuperscript{16} The return on stock $k$, $k = 1...N$, denoted by $x_k$, is normally distributed with mean 1 and variance $\sigma_k^2$. We sort the stocks based on variance, $0 < \sigma_1^2 < \sigma_2^2 < ... < \sigma_N^2$. Denote by $\alpha_k(i)$, $k = 1...N$, the fraction of wealth invested into the $k$-th security by fund manager $i$. Apart from the above, all other assumptions and notation are the same as in Section I.

It is easy to show, along the lines of proof of Proposition 1, that the industry-average absolute performance $\bar{R}$ is now given by

$$\bar{R} = \bar{r}^0 \left[ \sum_{k=1}^{N} \bar{\alpha}_k x_k + (1 - \sum_{k=1}^{N} \bar{\alpha}_k) \right],$$

\textsuperscript{15}The full text was available for some time at \url{http://banking.senate.gov/\_files/bogle.pdf}, but is no longer there.

\textsuperscript{16}If stocks are correlated, we can combine them to build $N$ uncorrelated portfolios, and then treat these portfolios as individual stocks.
where
\[
\tilde{\alpha}_k = \frac{\int_0^1 r^0(i)\alpha_k(i)di}{\tilde{r}_0}, \quad \tilde{r}_0 = \int_0^1 r^0(i)di.
\]

As before, the managers take \( \bar{R} \) as given when solving their maximization problems. In the interests of space, we do not present the best responses, but rather turn to an equilibrium. Let \( \sigma(r^0) \) denotes an equilibrium portfolio volatility of the fund with an interim performance \( r^0 \).

**Proposition 4** The function \( \sigma(r^0) \) is non-decreasing.

Proposition 4 reveals that our main result – funds with higher interim performance take on more risk – is robust to an \( N \)-stock extension.

### C Market Price of Risk and No Short-Sales

Suppose that the expected return on the risky stock, \( \mu \), is greater than 1, implying a positive market price of risk. All other assumptions are the same as in Section I. Proposition 5 describes the managers’ best responses in this case.

**Proposition 5** For given \( \tilde{\alpha} \), the solution of the optimization problem (6) is given by

\[
\alpha^*(i) = \begin{cases} 
1, & r^0(i) \geq \tilde{\alpha}\tilde{r}^0 - \frac{4\mu-1}{\sigma^2} \\
0, & \text{otherwise}
\end{cases}
\]  

From Proposition 5, winners (losers) hold only the risky (riskless) asset, and so our main result is unchanged. Compared to the base model, the risky stock is now more attractive for the managers. As a result, it is easy to show that now a larger share of the funds’ total wealth is invested into the risky stock. In the extreme case when \( \mu \) is very high, it is possible that in equilibrium all wealth is put into the stock.
Because the managers are risk-neutral and the flow-performance relationship is convex, we need to impose a short-sell constraint in order for the model to have a solution. However, it is not only for technical reasons that we make this assumption, but also because it seems appropriate in the context of mutual funds. Indeed, there are certain regulatory restrictions imposed – sometimes on a voluntarily basis – on institutional investors. For example, Almazan, Brown, Carlson, and Chapman (2004) document that 70% of mutual funds reported to the SEC that short-selling is not permitted under their investment policy. Among the remaining 30%, only 3% in fact engage in short-selling.

IV Conclusion

This paper investigates the validity of the so-called tournament hypothesis which is widely used in the empirical studies of the mutual fund industry. According to this hypothesis, interim winners are expected to decrease – and interim losers to increase – the volatility of their portfolios in the second half of the year. We solve a simple rational model of a mutual fund tournament and show the opposite: interim winners optimally choose a higher volatility of their portfolios than interim losers. Several recent empirical studies present evidence consistent with our model. However, based on the above, seemingly intuitive, tournament hypothesis the authors conclude that tournament behavior alone cannot explain their findings. Our results imply that this conclusion is unwarranted.
Appendix

Proof of Proposition 1.

Substituting (1) into (2), we get

\[ \bar{R} = \int_0^1 r^0(i)r^1(i) = \int_0^1 r^0(i)(\alpha(i)x + (1 - \alpha(i)) \, di \]

\[ = x \int_0^1 r^0(i)\alpha(i) \, di + \int_0^1 (1 - \alpha(i))r^0(i) \, di \]

\[ = \left( \frac{\int_0^1 r^0(i)\alpha(i) \, di}{\int_0^1 r^0(i) \, di} + \frac{\int_0^1 (1 - \alpha(i))r^0(i) \, di}{\int_0^1 r^0(i) \, di} \right) \int_0^1 r^0(i) \, di \]

\[ = r^0(\bar{\alpha}x + (1 - \bar{\alpha})) \]

where

\[ \bar{\alpha} = \frac{\int_0^1 r^0(i)\alpha(i) \, di}{\int_0^1 r^0(i) \, di} \quad \text{and} \quad r^0 = \int_0^1 r^0(i) \, di \]

Q.E.D.

Proof of Proposition 2.

Step 1. To characterize the optimal portfolios, we need the following well-known result: if \( v \) is normally distributed, \( v \sim N(\mu, \sigma^2) \), then \( E[\exp(v)] = \exp(\mu + \frac{1}{2}\sigma^2) \). Hence, maximizing \( E[\exp(v)] \) is equivalent to the maximization of \( \mu + \frac{1}{2}\sigma^2 \).

Step 2. The relative performance of fund \( i \) is given by

\[ R(i) - \bar{R} = r^0(i)(\alpha(i)x + 1 - \alpha(i)) - r^0(\bar{\alpha}x + (1 - \bar{\alpha})) \]

\[ = x(\alpha(i)r^0(i) - \bar{\alpha}r^0) + (1 - \alpha(i))r^0(i) - (1 - \bar{\alpha})r^0 \quad (A1) \]
Hence, it is normally distributed with constant mean

\[
E[x(\alpha(i)r^0(i) - \bar{\alpha}r^0) + (1 - \alpha(i))r^0(i) - (1 - \bar{\alpha})r^0] = (\alpha(i)r^0(i) - \bar{\alpha}r^0) + (1 - \alpha(i))r^0(i) - (1 - \bar{\alpha})r^0 = r^0(i) - \bar{r}^0
\]

and variance

\[
Var[x(\alpha(i)r^0(i) - \bar{\alpha}r^0) + (1 - \alpha(i))r^0(i) - (1 - \bar{\alpha})r^0] = \sigma^2(\alpha(i)r^0(i) - \bar{\alpha}r^0)^2.
\]

**Step 3.** From Steps 1 and 2, it follows that maximizing the expected terminal wealth, given in (5), is equivalent to

\[
\max_{\alpha(i), \alpha_i \in [0,1]} (\alpha(i)r^0(i) - \bar{\alpha}r^0)^2
\]

Because the objective function, denoted by \(u(\alpha(i))\), is convex the maximum is achieved at either \(\alpha_i = 0\) or \(\alpha_i = 1\). We have

\[
u(1) - u(0) = (r^0(i) - \bar{\alpha}r^0)^2 - (\bar{\alpha}r^0)^2 = (r^0(i))^2 - 2r^0(i)\bar{\alpha}r^0.
\]

Since \(r^0(i) > 0\) for all \(i\), the condition \(u(1) - u(0) > 0\) is equivalent to \(r^0(i) - 2\bar{\alpha}r^0 > 0\), and so (8) immediately follows.

\[Q.E.D.\]

**Proof of Proposition 3.**

First, we fix some value of \(\bar{\alpha} \in [0,1]\). From Proposition 2, fund \(k\) is indifferent between \(\alpha(k) = 1\) and \(\alpha(k) = 0\) if and only if

\[
r^0(k) = 2\bar{\alpha}r^0.
\]

(A2)
Since $r^0(i)$ is increasing in $i$, we have that the optimal policies of the managers are given by
\[ \hat{\alpha}(j) = \begin{cases} 
1, & \text{if } j \geq \tilde{k}, \\
0, & \text{otherwise}. 
\end{cases} \]

Aggregating as described in (7), we get
\[ \int_0^1 \frac{r^0(i)\alpha^*(j) \, dj}{r^0} = \frac{\int_0^1 r^0(i) \, di}{\bar{r}^0}. \]

In equilibrium, this quantity must be equal to $\bar{\alpha}$:
\[ \bar{\alpha} = \frac{\int_0^1 r^0(i) \, di}{\bar{r}^0}. \] (A3)

Solving A2 and A3 yields an equilibrium value of $bar\alpha$.

To see that an equilibrium value exists and is unique, notice that

(i) if $\bar{\alpha} = 0$ the left-hand side of (A3) is 0. When $\bar{\alpha} = 0$ all managers optimally choose $\hat{\alpha}(j) = 1$, and hence $k = 0$. So, the numerator and the denominator of the right-hand side in (A3) are equal to $\bar{r}^0$, and hence the right-hand side in of (A3) is 1.

(ii) if $\bar{\alpha} = 1$ the left-hand side in (A3) is 1. The right-hand side is less than 1, because $k > 0$.

(iii) the left-hand side of (A3) is increasing in $\bar{\alpha}$. From (A2), $k^*$ is increasing in $\bar{\alpha}$, hence the right-hand side of (A3) is decreasing in $\bar{\alpha}$.

Taken together, (i), (ii) and (iii) imply that there exists a unique solution to (A2) and (A3) such that $\bar{\alpha}^* \in (0, 1)$.

Q.E.D.
Proof of Proposition 4.

Consider two mutual funds \( i_1 \) and \( i_2 \), such that \( 0 \leq i_1 < i_2 \leq 1 \). Given that \( r^0(i) \) is increasing, fund \( i_2 \) has a higher interim return than fund \( i_1 \). To prove the Proposition, we show that for any values of \( \bar{\alpha}_k \) the outcome in which fund \( i_1 \) chooses a more volatile stock than fund \( i_2 \) cannot occur in an equilibrium. We prove this by contradiction.

First, note that a manager invests all his wealth into one asset as her objective function is convex. Suppose that managers \( i_1 \) and \( i_2 \) choose, respectively, stocks \( k_1 \) and \( k_2 \), such that \( 0 \leq k_2 < k_1 \leq N \). The bond corresponds to \( k = 0 \). Since manager \( i_1 \) optimally chooses stock \( k_1 \) versus \( k_2 \), we have

\[
\frac{1}{2} \sigma_{k_1}^2 (r^0(i_1) - \bar{\alpha}_{k_1} \bar{r})^2 + \frac{1}{2} \sigma_{k_2}^2 (\bar{\alpha}_{k_2} \bar{r})^2 > \frac{1}{2} \sigma_{k_1}^2 (r^0(i_1) - \bar{\alpha}_{k_2} \bar{r})^2 + \frac{1}{2} \sigma_{k_2}^2 (\bar{\alpha}_{k_1} \bar{r})^2. \quad (A4)
\]

Similarly, for manager \( i_2 \) we have

\[
\frac{1}{2} \sigma_{k_2}^2 (r^0(i_2) - \bar{\alpha}_{k_2} \bar{r})^2 + \frac{1}{2} \sigma_{k_1}^2 (\bar{\alpha}_{k_1} \bar{r})^2 > \frac{1}{2} \sigma_{k_2}^2 (r^0(i_2) - \bar{\alpha}_{k_1} \bar{r})^2 + \frac{1}{2} \sigma_{k_1}^2 (\bar{\alpha}_{k_2} \bar{r})^2. \quad (A5)
\]

Omitting simple algebra, (A4) is equivalent to

\[
\sigma_{k_1}^2 r^0(i_1) - 2 \sigma_{k_1}^2 \bar{\alpha}_{k_1} \bar{r} > \sigma_{k_2}^2 r^0(i_1) - 2 \sigma_{k_2}^2 \bar{\alpha}_{k_2} \bar{r}, \quad (A6)
\]

and (A5) is equivalent to

\[
\sigma_{k_2}^2 r^0(i_2) - 2 \sigma_{k_2}^2 \bar{\alpha}_{k_2} \bar{r} > \sigma_{k_1}^2 r^0(i_2) - 2 \sigma_{k_1}^2 \bar{\alpha}_{k_1} \bar{r}. \quad (A7)
\]

Finally, subtracting (A6) from (A7) yields:

\[
\sigma_{k_2}^2 (r^0(i_2) - r^0(i_1)) > \sigma_{k_1}^2 (r^0(i_2) - r^0(i_1)). \quad (A8)
\]
As we have assumed that \( r^0(i_2) > r^0(i_1) \), (A8) implies \( \sigma^2_{k_2} > \sigma^2_{k_1} \), which is contradiction. \( Q.E.D. \)

**Proof of Proposition 5.**

**Step 1.** From (A1), \((R(i) - \bar{R})\) is normally distributed with mean

\[
E[R(i) - \bar{R}] = [\alpha(i)r^0(i)(\mu - 1) - \bar{\alpha}r^0(\mu - 1) + r^0(i) - \bar{r}]
\]

and variance

\[
Var[R(i) - \bar{R}] = \sigma^2(\alpha(i)r^0(i) - \bar{\alpha}(i)r^0)^2.
\]

From (1), we have that \( E[R(i)] = r^0(i)(\alpha(i)(\mu - 1) + 1) \). Plugging the above expressions into the terminal wealth (5) and taking the expectation yields the maximization problem

\[
2r^0(i)\alpha(i)(\mu - 1) + \frac{1}{2}\sigma^2(\alpha(i)r^0(i) - \bar{\alpha}(i)r^0)^2 \rightarrow \max_{\alpha(i)\in[0,1]} \quad (A9)
\]

where, as in the proof of Proposition 2, we used the result on the expectation of a lognormal random variable. Again, denoting the objective function of manager \( i \) by \( u(\alpha(i)) \), we compare its value at \( \alpha(i) = 0 \) and \( \alpha(i) = 1 \):

\[
u(1) - u(0) = 2r^0(i)(\mu - 1) + \frac{1}{2}\sigma^2(r^0(i) - \bar{r}^0)^2 - [(\bar{\alpha}(i)r^0)^2
\]

\[
= 2r^0(i)(\mu - 1) + \frac{1}{2}\sigma^2 ((r^0(i))^2 - 2r^0(i)\bar{r}^0),
\]

After simple rearrangement, (11) follows. \( Q.E.D. \)
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