1 The Baumol–Tobin Model

Baumol and Tobin attempted to derive a demand–for–money function from first principles, and predict the sizes of the income and interest elasticities. They made the following assumptions:

Each person is paid $Y$ dollars per period (in a direct deposit to the bank). People have to decide on the optimal number of trips to the bank, $N$, to withdraw money over the course of the period. Assume that each trip they draw out the same amount of money, $Y/N$. The bank pays interest at rate $i$ on deposits. Cash of course earns no interest. Each trip to the bank costs the consumer a fixed cost in foregone time and inconvenience. Assume that this cost is $F$ dollars.

i) Solve for the optimal number of trips to the bank, $N^*$.

If consumers withdraw $Y/N$ dollars every time they go to the bank, then, assuming a smooth flow of consumption, on average they have $Y/(2N)$ dollars in their wallets. So on average they forgo $iY/2N$ dollars per period in lost interest earnings. The consumers’ problem is thus to minimize the total cost of going to the bank, which is the cost in terms of foregone interest plus the opportunity cost of their time, $F$. Hence, the problem is

$$\min_N FN + \frac{iY}{2N}.$$  

Obviously, the minimum is found by differentiating with respect to $N$ and setting the resultant first–order condition equal to zero:

$$F - \frac{iY}{2N^2} = 0.$$  

Thus,

$$N^* = \sqrt{\frac{iY}{2F}}. \quad (1)$$

ii) What is the average amount of money that the consumer holds? What are the elasticities of money demand with respect to income, $Y$, and the interest rate, $i$?

Plugging equation (1) back into the expression for average money holdings ($Y/(2N)$) yields an optimal average money holding of

$$M_{\text{avg}} = \sqrt{\frac{YF}{2i}}. \quad (2)$$

Recall that an elasticity can be expressed as a logarithmic derivative. Taking logs of equation (2),

$$\ln M_{\text{avg}} = \frac{1}{2} (\ln Y + \ln F) - \frac{1}{2} (\ln 2 + \ln i).$$

The income elasticity of money demand is then seen to be $\frac{1}{2}$, while the interest elasticity of money demand is $-\frac{1}{2}$. Notice that the “microfoundations” of this model imply very specific parameter values for our quantities of $L_Y(Y/M^4)$ and $L_r(r/M^4)$. Contrast this fact with the weaker assumptions we made in the IS–LM model, namely that $L_Y \geq 0$ and $L_r \leq 0$.

iii) Milton Friedman, a famous monetarist, championed the quantity theory of money: $M/P = aY$. What is the implication of this theory for the fiscal policy multiplier in the IS–LM model?

The quantity theory starts with the quantity equation $MV = PY$ (which is simply a definition) and assumes that velocity is constant at some level $\tilde{V}$. (Thus, above, $a = 1/\tilde{V}$.) In contrast with the more general Keynesian money demand relationship, there is no dependence on the interest rate in the quantity equation, thus $L_r = 0$. Graphically, we express the quantity equation with a vertical LM curve (since real output is simply proportional to real money balances).

We have shown that the fiscal policy multipliers can be written as:

$$\frac{dY}{dG} = \frac{L_r}{(1-C')L_r + I'LY},$$

$$\frac{dY}{dT} = \frac{-C'L_r}{(1-C')L_r + I'LY}.$$
about the income elasticity and the interest elasticity of money demand?

That opportunity cost varies linearly with income. How does this modification of the model alter its prediction?

In the introduction to his August 1956 article in the Review of Economics and Statistics, James Tobin writes why he considers this question:

One traditionally recognized source of demand for cash holdings is the need for transactions balances, to bridge the gaps in time between the receipts and the expenditures of economic units. By virtually common consent, this transactions demand for cash has been taken to be independent of the rate of interest.

The purpose of this paper is to [demonstrate that] ... [e]ven if there were unanimity and certainty that prevailing interest rates would continue unchanged indefinitely, so that no motive for holding cash other than transactions requirements existed, the demand for cash would depend inversely on the rate of interest. The reason is simply the cost of transactions between cash and interest-bearing assets.

Of course, as a “fiscalist,” Tobin is trying to develop a model with a non–zero interest elasticity of money demand (as opposed to the quantity equation). As one can confirm by analyzing the above equations for the fiscal–policy multipliers, the larger the elasticity of money demand (and $-\frac{1}{2}$ is rather large), the flatter the LM curve and the greater the effects of fiscal policy changes on the real level of output.

2 Testing the Baumol–Tobin Model

a) Empirical estimates of money demand place the income elasticity of money demand close to 1. Suppose you alter the Baumol–Tobin model to make the cost of a trip to the bank depend on income: instead of a cost, $F$, that is equal across consumers, assume it takes all consumers the same number of minutes to go to the bank, so that opportunity cost varies linearly with income. How does this modification of the model alter its prediction about the income elasticity and the interest elasticity of money demand?

We can easily modify our above results by letting $F = \beta Y$, where $\beta$ is some positive constant. Then, the expression for optimal money holdings (equation (2)) becomes

$$M_{avg} = \sqrt{\frac{\beta Y^2}{2i}} = Y \sqrt{\frac{\beta}{2i}}.$$ 

Then again taking logs of this expression for average money holdings, we find

$$\ln M_{avg} = \frac{1}{2} (\beta \cdot \ln 2) + \ln Y - \frac{1}{2} \ln i.$$ 

With this modification, the income elasticity of money demand is now equal to one, rather than $\frac{1}{2}$. (Note that this change has had no effect on the interest elasticity.)

b) People typically carry much less cash than the Baumol–Tobin model predicts, and go to the bank more often. Try the following exercise to see whether you can rationalize this finding.

i) Suppose your bank account pays you 5 percent interest per year. You spend $100 in cash each week. It takes 10 minutes of your time to make a trip to the bank. The opportunity cost of your time (your wage) is $12 per hour. According to the Baumol–Tobin model, how often should you go to the bank, and how much should you withdraw each time? (Be careful about the units of measurement.)

Equation (1) gives the formula for $N^*$ (the optimal number of trips to the bank). Here, $Y$ is ($100 a week $ \times 52$ weeks a year) = $5,200; i = .05; and $F$ = (10 minutes of time $\times 12$ an hour in wages) = $2 per trip to the bank. The optimal number of trips to the bank is therefore 8.06 times per year, or once every 45.3 days or so. The amount withdrawn each time is $5,200/8.06 = $645.16. Dividing this by two gives average money holdings, $322.58. Chances are, however, you have well under that amount in your wallet right now.

ii) In addition to the cost of foregone interest, a cost of holding money is that you might be robbed or lose your wallet. How high does the probability of this event need to be to make it rational for you to go to the bank once a week?

Denote the number of times you lose your wallet during the year as $z$, so that the yearly cost of of holding money is not only $i$ times money holdings, but $i + z$ times money holdings. Then, if we are to go to the bank once a week, $N^*$ will equal 52. Using the expression for $N^*$ above, we can solve for $z$:

$$52^\ast = \sqrt{5,200(0.05 + z)/(2 \cdot 2)}$$

so that $z = 2.03$ thefts per year, or about 0.039 thefts per week. According to these calculations, then, the percentage chance that you lose your wallet has to be about 3.9 percent in order for you to make one trip to the bank per week.
3 The Mundell–Tobin Effect

Consider the following full-employment IS-LM model:

**IS:** \( Y = C(Y - T, M/P) + I(r) + G. \)

**LM:** \( M/P = L(r + \pi, Y). \)

**AS:** \( Y = Y^*. \)

a.) Notice that investment depends on the real interest rate, \( r \), but money demand depends on the nominal interest rate, \( r + \pi \). In addition, consumption depends on real money balances. Explain why each of these assumptions is reasonable.

Investment depends on the real interest rate because the real interest rate measures the tradeoff between consumption now and consumption in the future. If you invest $1 now and get $1.15 a year from now, because we don’t know the rate of inflation during the period in which the investment is undertaken. Subtracting off the inflation rate \( \pi \) from the nominal interest rate \( i \) gives the real interest rate \( r \), which tells us how much our investments pay in real terms. This payoff, in turn, determines how willing people will be to suffer less consumption now in order to get more consumption later. Money demand depends on the nominal interest rate because the opportunity cost of holding money is what you give up by not holding bonds, i.e. the nominal interest rate. This fact is independent of the rate of inflation. Finally, real money balances might belong in the consumption function if real balances are an indicator of wealth, and if not holding bonds, i.e. the nominal interest rate. This fact is independent of the rate of inflation. (This would seem to be the case, given empirical evidence from the television show “Lifestyles of the Rich and Famous.”)

b.) Robert Mundell pointed out that, in this model, changes in the rate of inflation (\( \pi \)) affect the real interest rate. Hence, even though prices are fully flexible, money is not “super–neutral”; a change in the growth rate of money alters some real variables.

Solve for \( dr/d\pi \) in this model. What is its sign? Explain.

Totally differentiate the IS relation, holding \( dY \), \( dT \), \( dM \) and \( dG \) equal to zero (since they are equal to zero by assumption or don’t matter for \( dr/d\pi \)), to get

\[
0 = -C_{M/P}(M/P^2)dP + I' dr,
\]

where \( C_{M/P} \) is the derivative of consumption with respect to real money balances. Differentiating the LM relation, holding \( dM \) equal to zero, gives

\[
-(M/P^2)dP = L_i dr + L_i d\pi
\]

where \( L_i \) is the derivative of money demand with respect to the nominal interest rate. Placing these equations in matrix form yields

\[
\begin{bmatrix}
C_{M/P}(M/P^2) & -I' \\
(M/P^2) & L_i
\end{bmatrix}
\begin{bmatrix}
dP \\
dr
\end{bmatrix}
= \begin{bmatrix}
0 \\
-L_i d\pi
\end{bmatrix}
\]

Using Cramer’s rule to solve for \( dr/d\pi \) gives

\[
\frac{\left|\begin{array}{cc}
C_{M/P}(M/P^2) & 0 \\
(M/P^2) & -L_i d\pi
\end{array}\right|}{\left|\begin{array}{cc}
C_{M/P}(M/P^2)L_i & I'(M/P^2)
\end{array}\right|} = \frac{-C_{M/P}(M/P^2)L_i d\pi}{[C_{M/P}(M/P^2)L_i + I'(M/P^2)]}
\]

Dividing through by \( d\pi \) and cancelling the \( (M/P^2) \) terms gives

\[
\frac{dr}{d\pi} = \frac{-C_{M/P}L_i}{[C_{M/P}L_i + I']}
\]

The numerator in this expression is positive, since \( L_i \) is negative. The denominator is negative, since both the first term and the second term are negative. Hence, the entire expression is negative. Why would the real interest rate fall if inflation rises? The long–winded explanation goes something like this: A higher inflation rate means a higher nominal interest rate; it is like an unexpected tax on people’s money holdings. As a result they are less willing to hold the given stock of real balances. Since \( Y \) is fixed (by assumption), and \( M \) is not being changed, the only way to clear the money market is if the price level \( (P) \) rises. This increase in the price level reduces real balances \( (M/P) \) and brings the money market into balance. (Note: you should verify that \( dP/dr \) is positive.) By an increase in the price level, we mean a once–and–for–all jump in prices, apart from the sustained increase in prices represented by \( \pi \). Now, since real balances have fallen, consumption, \( C \), must also have fallen. Yet \( Y \) has to remain the same, and \( G \) hasn’t changed. So investment, \( I \), has to rise. But the only way that investment can rise is if the (real) interest rate falls. Hence, if we assume that \( Y \) is fixed and that \( C \) falls when real balances fall, we get an increase in inflation leading to a decrease in the real interest rate.
As you might have guessed, the long-winded explanation can be replaced by an IS–LM diagram, with either $i$ or $r$ on the vertical axis and the appropriate shifter variables added to the IS and LM curves. If you put $i$ on the vertical axis, inflation shifts out the IS curve, since with a given nominal interest rate, the real interest rate (which is important to the IS curve) falls as the rate of inflation rises. If you decide to put $r$ on the vertical axis, inflation shifts in the LM curve, since the nominal interest rate (which is important to the LM curve) rises for a given $r$ as $\pi$ rises. And for this problem, put $M/P$ as a shifter in the IS curve no matter how you label the vertical axis. The story above corresponds to a downward shift in the IS curve, since real balances will fall, leading to a drop in consumption that has to be made up by an increase in investment.

3.) Is this effect, which is called the Mundell effect (sometimes called the Mundell–Tobin effect) important quantitatively? That is, do you expect $dr/d\pi$ to be large or small?

In answering this question, you might need the following additional parameter values:

- Interest elasticity of investment = 0.8
- Interest elasticity of money demand = 0.1
- Income elasticity of money demand = 1.0
- Investment/GNP ratio = 0.15
- Marginal propensity to consume out of income = 0.5
- (Outside) money–GNP ratio: 0.1
- Marginal propensity to consume out of wealth: 0.05

To answer this question, write $dr/d\pi$ as

$$\frac{-1}{1 + \frac{I'}{L_iC_{M/P}}}. \frac{(r/I)(M^d/r)(Y/M^d)}{(Y/I)}$$

Now multiply this expression by $(r/I)(M^d/r)(Y/M^d) = (Y/I)$ so that the second term in the denominator becomes

$$[I'(r/I)] \left[ (M^d/r)/L_i \right] \left[ 1/C_{M/P} \right] \left[ Y/M^d \right],$$

or

$$[-0.8][-1/0.1][1/0.05][1/0.10] = 1.600.$$

Since $(Y/I) = 1/0.15$, the entire Mundell–Tobin effect is approximately equal to $(-1/0.15)/(1/0.15 + 1.600) = -0.00415$. This puny effect isn’t surprising, since the low interest elasticity of money demand means people don’t care much about the nominal interest rate when deciding on their money balances (i.e. the price level is not going to have to jump much in order to clear the money market), and the very low marginal propensity to consume out of wealth, $C_{M/P} = 0.05$, means that wealth or real balances don’t matter much in the consumption function (i.e. the IS curve doesn’t shift down much). Hence the overall effect is small.

4 The Lucas Critique and the Phillips Curve

Suppose an economy is governed by an expectations-augmented Phillips curve:

$$u = u^* - \alpha(\pi - E\pi)$$

where

- $u$ = unemployment,
- $u^*$ = the natural rate of unemployment,
- $\pi$ = inflation,
- $E\pi$ = expected inflation.

a) What is the time path of unemployment if
   i.) inflation is always zero?

   If inflation is always zero, expected inflation is always equal to zero, and $\pi - E\pi$ is always equal to zero. Hence, unemployment is always at the natural rate.

   ii.) if inflation is a constant five percent?

   In this case, expected inflation is always equal to five percent, so unemployment is always at the natural rate, just as before. It doesn’t matter what rate of inflation characterizes the economy; if inflation is constant, expected inflation will always equal that value and the rate of unemployment will stay at the natural rate.

b) Suppose inflation is random, uniformly distributed between zero and ten percent.

   i.) What is expected inflation?
The formula for expected inflation in this case is

\[ E(\pi) = \int_{0}^{10} \pi f(\pi) d\pi, \]

where \( f(\pi) \) is the probability distribution of \( \pi \). Since we are told that inflation is uniformly distributed between zero and ten, \( f(\pi) = 1/10 \) and \( E(\pi) = 5 \). This makes sense, since if inflation varies uniformly between 0 and 10, we would “expect” it to be 5 percent on average.

ii.) What does the observed Phillips Curve look like?

The Phillips Curve is usually graphed with inflation on the vertical axis and unemployment on the horizontal axis. So we’ll plug 5 into our expectations–augmented Phillips curve as the value of \( E(\pi) \) and put inflation on the left–hand side of the equation:

\[ u = u^* - \alpha (\pi - 5) \]
\[ u^* - u = \alpha (\pi - 5) \]
\[ \pi = (1/\alpha)(u^* - u) + 5, \]

so we can draw the graph:

![Phillips Curve Graph](attachment:image.png)

c) Suppose the stochastic process generating inflation changes to be uniform from 5 to 15 percent. What will the observed Phillips Curve look like now?

Following the same steps as above, we can show that expected inflation equals 10 percent, so we can write the Phillips Curve relation as

\[ \pi = (1/\alpha)(u^* - u) + 10. \]

The graph becomes:

![Phillips Curve Graph](attachment:image.png)

The Phillips Curve has shifted up to compensate for the higher level of expected inflation.

d) What menu of inflation–unemployment combinations does this economy offer to policy makers? What menu might policy makers think they have available?

This economy offers policy–makers a tradeoff between inflation and unemployment in the short run, not the long run, because the long–run Phillips Curve is vertical. Should policy makers want to change the stochastic process generating inflation, to move the economy northwest along the first Phillips Curve graphed, people would eventually update their expectations of inflation so that the Phillips Curve would shift up. Unemployment would gravitate back to its natural level.

5 Rational Expectations

Money demand equation:

\[ m_t - p_t = \gamma + \alpha (E_{t+1}p_t - p_t) + u_t. \]
Money supply equation:

\[ mt = \beta_0 + \beta_1 p_{t-1} + e_t. \]  

(4)

Substituting \( m_t \) from (4) to (3) yields

\[ \beta_0 + \beta_1 p_{t-1} + e_t = \gamma + \alpha E_t p_{t+1} + (1 - \alpha) p_t + u_t, \]  

(5)

Let’s guess that there is a solution of the form

\[ p_t = \phi_0 + \phi_1 p_{t-1} + \phi_2 (u_t - e_t). \]

Therefore,

\[ E_t p_{t+1} = \phi_0 + \phi_1 p_t = \phi_0 + \phi_1 (\phi_0 + \phi_1 p_{t-1} + \phi_2 (u_t - e_t)). \]

Substituting to (5), we obtain

\[ \beta_0 + \beta_1 p_{t-1} + e_t = \gamma + \alpha (\phi_0 + \phi_1 (\phi_0 + \phi_1 p_{t-1} + \phi_2 (u_t - e_t))) + (1 - \alpha) (\phi_0 + \phi_1 p_{t-1} + \phi_2 (u_t - e_t)) + u_t. \]

This is an identity, so the following equations hold:

\[ \begin{align*}
0 &= \alpha \phi_1 \phi_2 + (1 - \alpha) \phi_2 + 1 \implies \phi_2 = -\frac{1}{(1 - \alpha) + \alpha \phi_1}(u_t - e_t), \\
\beta_0 &= \gamma + \alpha \phi_0 + \alpha \phi_1 \phi_0 + (1 - \alpha) \phi_0 \implies \phi_0 = \frac{\beta_0 - \gamma}{1 + \alpha \phi_1}, \\
\beta_1 &= \alpha \phi_1^2 + (1 - \alpha) \phi_1
\end{align*} \]

(1)

Solving square equation one can get

\[ \phi_1 = \frac{-(1 - \alpha) + \sqrt{(1 - \alpha)^2 + 4 \alpha \beta_1}}{2 \alpha}, \quad \phi_2 = \frac{-(1 - \alpha) - \sqrt{(1 - \alpha)^2 + 4 \alpha \beta_1}}{2 \alpha}. \]

These are two possible values for the coefficient attached to \( p_{t-1} \).

The first value of \( \phi_1 \) leads to the stationary solution, while the second leads to nonstationary one (or no solution at all). To illustrate this, consider the case when \( \beta_1 = 0 \). Then \( \phi_1 = -(1 - \alpha)/\alpha \) and equation \( \alpha \phi_1 \phi_2 + (1 - \alpha) \phi_2 + 1 = 0 \) does not have solutions.